Investigation • Multiply and Conquer

Name ______________________________ **Period** __________ **Date** ______________

You can easily guess the value of M in the proportion $\frac{2}{3} = \frac{M}{6}$. In this investigation you'll examine ways to solve a proportion for an unknown number when guessing is not easy. It's hard to guess the value of M in the proportion $\frac{M}{19} = \frac{56}{133}$.

Step 1 Multiply both sides of the proportion $\frac{M}{19} = \frac{53}{133}$ by 19. Why can you do this? What does M equal?

Step 2 For each equation, choose a number to multiply both ratios by to solve the proportion for the unknown number. Then multiply and divide to find the missing value.

a. $\frac{p}{12} = \frac{132}{176}$ **b.** $\frac{21}{35} = \frac{Q}{20}$

c. $\frac{L}{30} = \frac{30}{200}$ **d.** $\frac{130}{78} = \frac{n}{15}$

Step 3 Check that each proportion in Step 2 is true by replacing the variable with your answer.

Step 4 In each equation in Step 2, the variables are in the numerator. Write a brief explanation of one way to solve a proportion when one of the numerators is a variable.

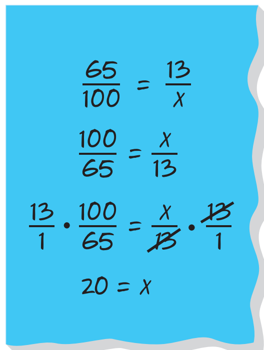
Step 5 The proportions you solved in Step 2 have been changed by switching the numerators and denominators. That is, the ratio on each side has been *inverted*. (You may recall that inverted fractions, like $\frac{p}{12}$ and $\frac{12}{p}$ are called *reciprocals*.) Do the solutions from Step 2 also make these new proportions true?

a. $\frac{12}{p} = \frac{176}{132}$ **b.** $\frac{35}{21} = \frac{20}{Q}$ **c.** $\frac{30}{L} = \frac{200}{30}$ **d.** $\frac{78}{130} = \frac{15}{n}$

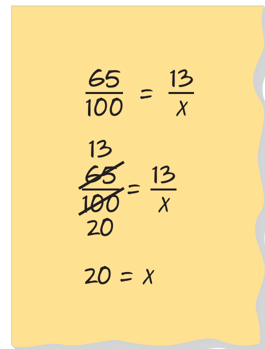
Step 6 How can you use what you just discovered to help you solve a proportion that has the variable in the denominator, such as $\frac{20}{135} = \frac{12}{k}$? Why does this work? Solve the equation.

Step 7 There are many ways to solve proportions. Here are three student papers each answering the question “13 is 65% of what number?” What are the steps each student followed? What other methods can you use to solve proportions?

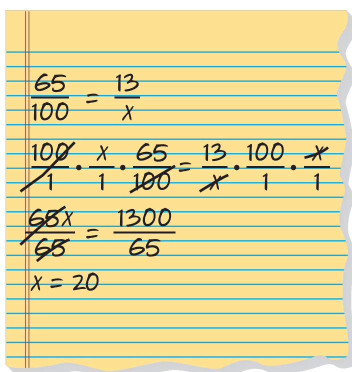
a.



b.



c.



Investigation • Fish in the Lake

Name ______________________ **Period** __________ **Date** ______________

You will need: a paper bag, white beans, red beans

In this investigation you'll **simulate** the capture-recapture method and examine how it works.

The bag represents a lake, the white beans are the untagged fish in the lake, and the red beans will replace white beans to represent tagged fish. Your objective is to estimate the total number of fish in the lake.

Step 1 Reach into the lake and remove a handful of fish to tag. Count and record the number of fish you removed. Replace these fish (white beans) with an equal number of tagged fish (red beans). Return the tagged fish to the lake. Set aside the extra beans.

Step 2 Allow the fish to mingle (seal the bag and shake it). Again remove a handful of fish, count them all, and count the number of tagged fish. In this table record those counts and the ratio of tagged fish to total fish in the sample.

Tagging Simulation

Sample number	Number of tagged fish	Total number of fish	Ratio of tagged fish to total fish
1			
2			
3			
4			
5			

You have taken one sample by randomly capturing some of the fish. You could use this sample to estimate the number of fish in the lake, but by taking several samples, you will get a better idea of the ratio of tagged fish to total fish in the lake. Replace the fish, mix them, and repeat the sampling process four times, filling in a row of your table each time.

Step 3 Choose a representative ratio for the five ratios. Explain how you decided this was a representative ratio.

Step 4 If you mixed the fish well, should the fraction of tagged fish in a sample be nearly the same as the fraction of tagged fish in the lake? Why or why not?

Step 5 Write and solve a proportion to find the number of fish in the lake. (About how many beans are in your bag?) Why is this method called capture-recapture? How accurate are predictions using this method? Why?

Investigation • Fish in the Lake With Sample Data

Name ____________________ Period __________ Date ______________

In this investigation you'll **review** the capture-recapture method and examine how it works, using sample data rather than data you have collected.

In the experiment, a paper bag was used to represent a lake and white beans to represent the untagged fish in the lake. Red beans were used to replace white beans to represent tagged fish. The objective is to estimate the total number of fish in the lake.

Step 1 A handful of fish (white beans) was removed from the lake (bag). The removed fish were counted and recorded in the table in the column "Number of tagged fish." Those fish (white beans) were replaced with an equal number of tagged fish (red beans) that were returned to the bag. (The removed white beans were set aside.)

Step 2 The fish were allowed to mingle (the sealed bag was shaken) and another sample was removed. All of the fish in the sample and the tagged fish in the sample were counted and recorded in the table before being returned to the lake. The process was repeated until results for five samples were recorded. Complete the table by calculating the ratio of tagged fish to total fish for each sample.

Tagging Simulation

Sample number	Number of tagged fish	Total number of fish	Ratio of tagged fish to total fish
1	8	48	
2	24	102	
3	16	86	
4	17	67	
5	16	75	

Step 3 Choose a representative ratio for the five ratios. Explain how you decided this was a representative ratio.

Step 4 If you mixed the fish well, should the fraction of tagged fish in a sample be nearly the same as the fraction of tagged fish in the lake? Why or why not?

Step 5 Write and solve a proportion to find the number of fish in the lake. (About how many beans are in your bag?) Why is this method called capture-recapture? How accurate are predictions using this method? Why?

Investigation • Converting Centimeters to Inches

Name ______________________________ **Period** __________ **Date** ______________

You will need: a yardstick or tape measure, a meterstick or metric tape measure

In this investigation you will find a ratio to help you convert inches to centimeters and centimeters to inches. Then you will use this ratio in a proportion to convert some measurements from the system standard in the United States to measurements in the metric system, and vice versa.

Step 1 Measure the length or width on each of six different-sized objects, such as a pencil, a book, your desk, or your calculator. For each object, record the inch measurement and the centimeter measurement in this table.

Object	Measurement in inches	Measurement in centimeters

Step 2 Enter the measurements in inches into your calculator's list L1 and the measurements in centimeters into list L2. Into list L3 enter the ratio of centimeters to inches, $\frac{L2}{L1}$, and let your calculator fill in the ratio values. [▸ See **Calculator Note 1K: Formula-Generated Lists.** ◂]

Step 3 How do the ratios of centimeters to inches compare for the different measurements? If one of the ratios is much different from the others, recheck your measurements.

Investigation • Converting Centimeters to Inches (continued)

Step 4 Choose a single representative ratio of centimeters to inches. Write a sentence that explains the meaning of this ratio.

Step 5 Using your ratio, set up a proportion and convert each length.

a. 215 centimeters = x inches

b. 1 centimeter = x inches

c. 1 inch = x centimeters

d. How many centimeters high is a doorway that measures 80 inches?

Step 6 Using your ratio, set up a proportion and solve for the requested value.

a. y centimeters = x inches. Solve for y.

b. c centimeters = i inches. Solve for i.

Investigation • Ship Canals

Name ______________________ **Period** __________ **Date** ____________

You will need: a ruler

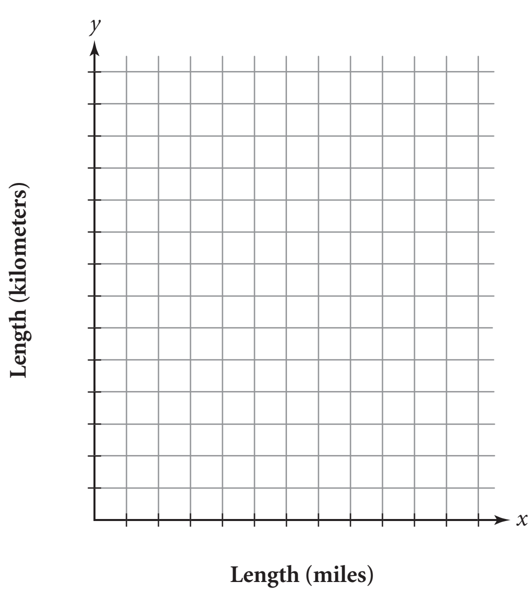
In this investigation you will use data about canals to draw a graph and write an equation that states the relationship between miles and kilometers. You'll see several ways of finding the information that is missing from this table.

Ship Canals

Canal	Length (miles)	Length (kilometers)
Albert (Belgium)	80	120
Alphonse XIII (Spain)	53	85
Houston (Texas)	50	81
Kiel (Germany)	62	99
Main-Danube (Germany)	106	171
Moscow-Volga (Russia)	80	129
Panama (Panama)	51	82
St. Lawrence Seaway (Canada/U.S.)	189	304
Suez (Egypt)	101	
Trollhatte (Sweden)		87

(The Top 10 of Everything 1998, p. 57)

Step 1 Carefully scale these coordinate axes for the data in the table. Let x represent the length in miles and y represent the length in kilometers. Plot points for the first eight coordinate pairs.



Investigation • Ship Canals (continued)

Step 2 What pattern or shape do you see in your graph? Connect the points to illustrate this pattern. Explain how you could use your graph to approximate the length *in kilometers* of the Suez Canal and the length *in miles* of the Trollhätte Canal.

Step 3 On your calculator, make a plot of the same points and compare it to your hand-drawn plot. Use list L1 for lengths in miles and list L2 for lengths in kilometers. [▶ See **Calculator Note 1F: Scatter Plots** to review this type of plot. ◀]

Step 4 Use list L3 to calculate the ratio $\frac{L2}{L1}$. [▶ See **Calculator Note 1K: Formula-Generated Lists** to review using lists to calculate this way. ◀] Explain what the values in list L3 represent. If you round each value in list L3 to the nearest tenth, what do you get?

Step 5 Use the rounded value you got in Step 4 to find the length in kilometers of the Suez Canal. Could you also use your result to find the length in miles of the Trollhätte Canal?

The number of kilometers is the same in every mile, so the value you found is called a **constant.**

Step 6 How can you change x miles to y kilometers? Using variables, write an equation to show how miles and kilometers are related.

Investigation • Ship Canals (continued)

Step 7 Use the equation you wrote in Step 6 to find the length in kilometers of the Suez Canal and the length in miles of the Trollhätte Canal. How is using this equation like using a rate?

Step 8 Graph your equation on your calculator. [▶ See **Calculator Note 1J: Equations** to review graphing equations. ◀] Compare this graph to your hand-drawn graph. Why does the graph go through the origin?

Step 9 Trace the graph of your equation. [▶ See **Calculator Note 1J: Equations** to review tracing equations. ◀] Approximate the length in kilometers of the Suez Canal by finding when x is approximately 101 miles. Trace the graph to approximate the length in miles of the Trollhätte Canal. How do these answers compare to the ones you got from your hand-drawn graph?

Step 10 Use the calculator's table function to find the missing lengths for the Suez Canal and the Trollhätte Canal. [▶ See **Calculator Note 2A: Tables** to learn about the table function. ◀]

Step 11 In this investigation you used several ways to find missing values—approximating with a graph, calculating with a rate, solving an equation, and searching a table. Write several sentences explaining which of these methods you prefer and why.

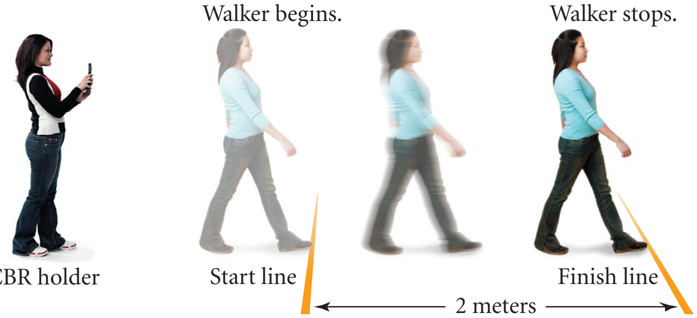
Investigation • Speed versus Time

Name ______________________________ **Period** __________ **Date** ____________

You will need: one motion sensor

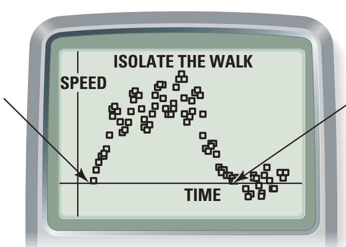
In this investigation you will explore the relationship between a walker's speed and the time it takes to cover a fixed-length course.

Step 1 Set up the course by marking a starting line and a finish line 2.00 m apart.



Step 2 Perform the activity as described in the Procedure Note.

Step 3 On the calculator, press ENTER to download the data. Isolate the part of the graph that shows the walk by moving the cursor to the right to just where the walk began. (Remember, the graph shows speed versus time.) Press ENTER. Move the cursor until it returns to the x-axis at the end of the walk. Press ENTER again. Now you should see just the walk data. If it is correct, press 1.



Procedure Note

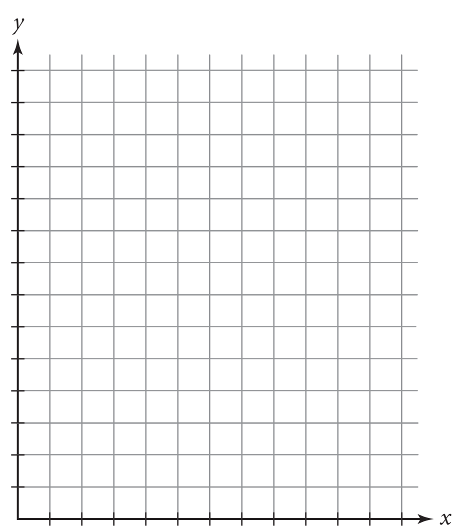
1. Download the INVERSE program to your graphing calculator. [▶ See **Calculator Note 2B: INVERSE Program.**◀]
2. Run INVERSE, and follow the directions that appear on the calculator screen. To begin, the walker stands at the start line, and the CBR holder stands 1 m behind the start line, facing the walker.
3. The CBR holder presses the trigger of the CBR. The CBR will collect data for 10 s. Approximately 1 s after the CBR starts, the walker walks to the finish line and comes to a stop. The walker waits until the 10 s are complete.

Step 4 Your calculator will now display the walk number, the total time for the walk, and the average speed of the walker. Record these data in the table.

Walk number	Total time (s)	Average speed (m/s)
1		
2		
3		
4		
5		
6		

Step 5 Press ENTER, and trade jobs among the group members. Repeat Steps 1–4 five times, to collect data for six walks. Try to do two different slow walks, two different medium walks, and two different fast walks. Record those data in the table.

Step 6 When the program is complete, enter the six (*total time, average speed*) data points into lists in each group member's calculator. Create a graph that shows the data and both axes.



Investigation • Speed versus Time (continued)

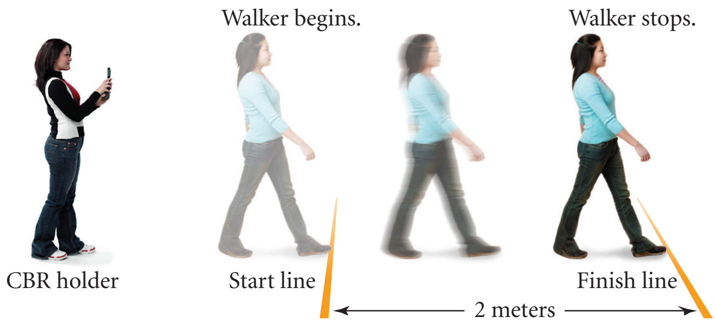
Step 7 Find an equation in the form $y = \frac{a}{x}$ that is a good model for the relationship between total time and average speed. Experiment with different values of a until you find a curve that looks like a good fit for the data.

Step 8 What does the value of a found in Step 7 have to do with the experiment? What kind of units does it have?

Name ________________ **Period** ________ **Date** ________

In this investigation you will explore the relationship between a walker's speed and the time it takes to walk 2.00 meters.

Step 1 The course was set up by marking a starting line and a finish line 2.00 m apart.



Step 2 The activity was performed as described in the Procedure Note.

Step 3 Data collected by a motion sensor was downloaded to a graphing calculator.

Step 4 The calculator displayed the walk number, the total time for the walk, and the average speed of the walker.

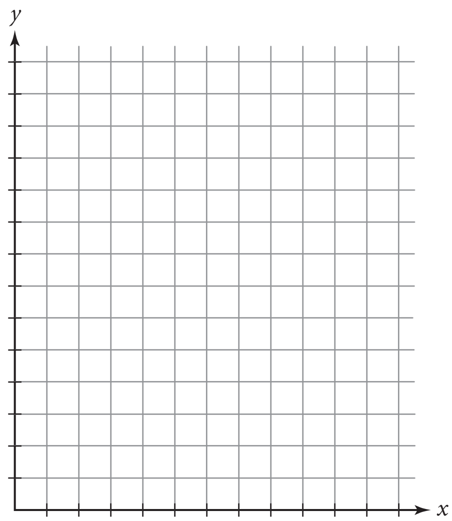
Step 5 Steps 1–4 were repeated five more times, to collect data for six walks, shown in this table. Complete the table by determining the average speed for each walk.

Walk number	Total time (s)	Average speed (m/s)
1	6.2	
2	8	
3	2.3	
4	1.8	
5	4.7	
6	5.9	

Procedure Note

1. Download the INVERSE program to your graphing calculator. [▶ See **Calculator Note 2B: INVERSE Program.**◀]
2. Run INVERSE, and follow the directions that appear on the calculator screen. To begin, the walker stands at the start line, and the CBR holder stands 1 m behind the start line, facing the walker.
3. The CBR holder presses the trigger of the CBR. The CBR will collect data for 10 s. Approximately 1 s after the CBR starts, the walker walks to the finish line and comes to a stop. The walker waits until the 10 s are complete.

Step 6 Enter the six (*total time, average speed*) data points into lists in each group member's calculator. Create a graph that shows the data and both axes.



Step 7 Find an equation in the form $y = \frac{a}{x}$ that is a good model for the relationship between total time and average speed. Experiment with different values of *a* until you find a curve that looks like a good fit for the data.

Step 8 What does the value of *a* found in Step 7 have to do with the experiment? What kind of units does it have?

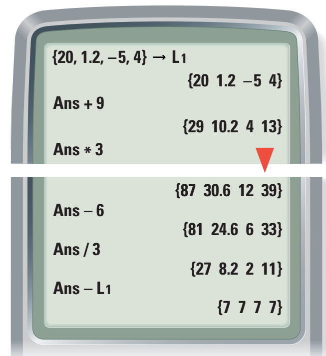
Investigation • Number Tricks

Name ______________________________ **Period** __________ **Date** ______________

Try this trick: Each member of your group should think of a different number from 1 to 25. Add 9 to it. Multiply the result by 3. Subtract 6 from the current answer. Divide this answer by 3. Now subtract your original number. Compare your results.

Do you think the result will be the same regardless of the number you start with? Do you think it would still work even if you chose a decimal number, a fraction, or a negative number? One way to answer these questions is to use a list of numbers instead of just a single starting value.

Step 1 Enter a list of at least four different numbers into the calculator home screen and store this list in list L1. In the example at right, the list is {20, 1.2, −25, 4}, but you should try different numbers. Perform the operations on your own starting numbers. The last operation is to subtract your original number.



Step 2 Explain how the last operation is different from the others.

Step 3 Number tricks like this work because certain operations, such as multiplication and division, get “undone” in the course of the trick. Which step undoes Ans · 3?

Step 4 One way to analyze what is happening in a number trick is to translate the steps of the trick into an algebraic expression. Return to the description at the beginning of this investigation and write an algebraic expression using x to represent your starting number.

Description	Expression
Starting value	x
Add 9.	$x + 9$

Investigation • Number Tricks (continued)

Step 5 You can use the method below to help you figure out why any number trick works. The symbol +1 represents one positive unit. You can think of n as a variable or as a container for different unknown starting numbers. Complete the Description column by writing the steps in this new number trick.

Stage	Picture	Description	Expression
1	n	Pick a number.	
2	n +1 +1 +1		
3	n n +1 +1 +1 +1 +1 +1		
4	n n +1 +1		
5	n +1		
6	+1	Subtract the original number.	
7	+1 +1 +1		

Step 6 Complete the Expression column by writing an algebraic expression for each step in the trick.

Step 7 Evaluate the final expression in Step 6 using a list of starting numbers. What is the result? Explain why this happens.

Step 8 Invent your own number trick that has at least five stages. Test it on your calculator with a list of at least four different numbers to make sure all the answers are the same. When you're convinced the number trick is working, try it on the other members of your group.

Investigation • Just Undo It!

Name ______________________ **Period** __________ **Date** ______________

Step 1 Choose a secret number. Now choose four more nonzero numbers and in any random order add one of them, multiply by another, subtract another, and divide by the final number. Record in words what you did and your final result on a blank sheet of paper. (For example, "I took my secret number, divided by 4, added 7, multiplied by 2, and subtracted 8. The result was 28.") Do not record your secret number. Trade papers with another student.

Step 2 Use the description on the paper given to you to complete the description, sequence, and expression columns in the table below, as shown in the sample table.

Description	Sequence	Expression		
Picked a number.	?	x		
Divided by 4.	Ans / 4	$\frac{x}{4}$		
Added 7.	Ans + 7	$\frac{x}{4} + 7$		
Multiplied by 2.	Ans · 2	$2\left(\frac{x}{4} + 7\right)$		
Subtracted 8.	Ans − 8	$2\left(\frac{x}{4} + 7\right) - 8$		

Description	Sequence	Expression	Undo	Result
Picked a number.	?	x		

Investigation • Just Undo It! (continued)

Step 3 Now fill in the Undo column in your table, listing the operations needed to undo each step. (See this sample table for guidance.)

Description	Sequence	Expression	Undo	
Picked a number.	?	x		
Divided by 4.	Ans / 4	$\frac{x}{4}$	$\cdot$ (4)	
Added 7.	Ans + 7	$\frac{x}{4} + 7$	− (7)	
Multiplied by 2.	Ans • 2	$2\left(\frac{x}{4} + 7\right)$	/ (2)	
Subtracted 8.	Ans − 8	$2\left(\frac{x}{4} + 7\right) - 8$	+ (8)	

Step 4 In the fifth column of your table, put the final result in the bottom right cell. Then work up the table from the bottom, undoing each operation as shown, to discover the original number, as shown here. Was this the secret number? (In this example the final result was 28 and the original secret number was 44.)

Description	Sequence	Expression	Undo	Result
Picked a number.	?	x		44
Divided by 4.	Ans / 4	$\frac{x}{4}$	$\cdot$ (4)	11
Added 7.	Ans + 7	$\frac{x}{4} + 7$	− (7)	18
Multiplied by 2.	Ans • 2	$2\left(\frac{x}{4} + 7\right)$	/ (2)	36
Subtracted 8.	Ans − 8	$2\left(\frac{x}{4} + 7\right) - 8$	+ (8)	28

Many equations can be solved using a table by undoing each operation, following these steps.

1. Complete the description column using the order of operations.
2. Complete the undo column.
3. Finally, work up from the bottom of the table to solve the equation.

You can check your solution to an equation by substituting the solution into the original equation and evaluating to check that you get a true statement.

Investigation • Just Undo It! (continued)

Study this example. Next you will create your own table to solve an equation.

Equation: $\frac{3 + 2(x - 4)}{5} + 6 = 11$		
Description	**Undo**	**Result**
Pick x.		15
$-$ (4)	$+$ (4)	11
• (2)	/ (2)	22
$+$ (3)	$-$ (3)	25
/ (5)	• (5)	5
$+$ (6)	$-$ (6)	11

Step 5 Solve this equation using a table: $7 + \frac{x - 3}{4} = 42$. Check your solution.

Equation:		
Description	**Undo**	**Result**
Pick x.		

Step 6 Write a few sentences explaining why this method works to solve an equation.